

Math 7760 – Homework 7 – Due: November 4, 2022

Practice Problems:

Problem 1. Describe the rank function, closure operator, and lattice of flats of M/e .

Problems to write up:

Problem 2. Prove that $\mathcal{M}(K_{3,3})^*$ is not graphic.

Problem 3. Prove directly, without using matroid duality, that M/e is representable over a field \mathbb{F} whenever M is [hint: obtain a representation of M/e from a representation of M by projecting all columns not corresponding to e onto the hyperplane orthogonal to e].

Problem 4. Let G be a directed graph with vertex set V and directed edge set E and let $U(G)$ be the undirected graph obtained from G by dropping edge orientations. For each cycle C of $U(G)$ let C^+ and C^- denote the elements of $\{+, -, 0\}^E$ obtained as follows. Set $C_e^+ = C_e^- = 0$ for each $e \in E \setminus C$. Choose a cyclic ordering of the vertices v_1, \dots, v_k of C . For each $e \in C$, set $C_e^+ = +$ if e is directed from v_i to v_{i+1} and $-$ otherwise. Prove that this collection of sign vectors is the set of signed circuits of an oriented matroid. What are the signed vectors?