

## Math 7760 – Homework 5 – Due: October 10, 2022

### Practice Problems:

**Problem 1.** Let  $G$  be a graph. Describe the closure operator of  $\mathcal{G}$  in graph-theoretic terms.

**Problem 2.** Oxley section 1.4 problem 6.

### Problems to write up:

**Problem 3.** Prove that a graph with  $n$  vertices,  $c$  connected components, and at least  $n - c + 1$  edges has a cycle. Then let  $G$  be a graph with edge set  $E$  and show that its matroid  $\mathcal{M}(G)$  has the following rank function

$$\rho(S) = |V(S)| - c(S)$$

where  $V(S)$  denotes the set of vertices of  $G$  that are incident to some edge in  $S$  and  $c(S)$  denotes the number of connected components of the graph on vertex set  $V(S)$  and edge set  $S$ .

**Problem 4.** Let  $E$  be a finite set and let  $\mathcal{H} \subseteq 2^E$ . Prove that  $\mathcal{H}$  is the set of hyperplanes of a matroid if and only if

- (1)  $E \notin \mathcal{H}$ ,
- (2) if  $H_1, H_2 \in \mathcal{H}$  with  $H_1 \subseteq H_2$ , then  $H_1 = H_2$ , and
- (3) if  $H_1 \neq H_2 \in \mathcal{H}$  and  $e \notin H_1 \cup H_2$ , then there exists  $H \in \mathcal{H}$  such that  $H \supseteq (H_1 \cap H_2) \cup e$ .